Abdullah *et al. BMC Medical Informatics and Decision Making (2025) 25:122*
https://doi.org/10.1186/s12911-025-02946-4

BMC Medical Informatics and Decision Making

RESEARCH

Open Access

An intelligent multi-attribute decision-making system for clinical assessment of spinal cord disorder using fuzzy hypersoft rough approximations

Muhammad Abdullah[1], Khuram Ali Khan[1*] and Atiqe Ur Rahman[2]

Abstract

The data for diagnosing spinal cord disorder (SCD) are complex and often confusing, making it difficult for established diagnostic techniques to yield reliable results. This issue frequently necessitates expensive testing to get an accurate diagnosis. However, the diagnostic process can be enhanced by integrating theoretical frameworks that resemble fuzzy sets, which better manage complexity and uncertainty. This integration reduces the frequency of expensive diagnostic procedures, improving the effectiveness of decision-making. The goal of this work is to present lower and upper approximations for fuzzy hypersoft sets, which employ multi-argument-based parameters to improve the traditional lower and upper approximations of fuzzy sets and soft sets. An intelligent mechanism for decision assistance is established by proposing a robust algorithm, that is based on the proposed approximations. To validate the proposed algorithm, a prototype case study for the clinical diagnosis of SCD is discussed. The criteria are further refined by using pertinent sub-criteria, such as functional ability, imaging data, and neurological status criteria. Medical professionals would find the suggested approximations to be a very helpful tool as the results indicate that they could greatly improve diagnosis. This study contributes to the field of medical diagnostics by providing a sophisticated multi-criteria analytical tool that can manage the complexity and inherent ambiguity of SCD diagnosis.

Keywords Clinical assessment, Medical informatics, Spinal cord disorder, Multi-attribute decision-making, Rough approximations, Fuzzy hypersoft set

Introduction

Spinal cord disorder (SCD), a devastating and severely incapacitating condition, can be brought on by a variety of external factors, including traffic accidents, high-altitude falls, physical trauma, sports injuries, and other traumatic events [1]. SCD causes limb dysfunction, including loss of motor and sensory abilities, as well as paralysis. Autonomic nerve dysfunction, which includes bowel and urine incontinence, may also be the outcome. Both directly and indirectly, SCD can also result in severe complications like central neuralgia, lower limb deep vein thrombosis, bedsores, lung and urinary tract infections, and limb abnormalities. As a result, those who have suffered from SCD and become paralyzed place a heavy burden on society, their families, and themselves because they are unable to work, require extensive medical care, and cannot afford expensive rehabilitation. The incidence of SCD has been estimated to be between 10.4

*Correspondence:
Khuram Ali Khan
khuram.ali@uos.edu.pk
[1] Department of Mathematics, University of Sargodha, Sargodha 40100, Pakistan
[2] Department of Mathematics, University of Management and Technology, Lahore 54000, Pakistan

and 83 cases per million annually worldwide [2, 3]. In US, the incidence of SCD is 54 cases per million, with approximately 17,810 new cases annually [4–6]. In China, the situation is even more dire, with an annual incidence of approximately 23.7 to 60.6 cases per million [7, 8]. Because of the extremely limited regenerative capacity and irreversible damage to neuronal systems, effective treatments for SCD remain elusive in clinical practice despite tremendous advances in understanding the etiology and secondary injury processes of the condition. One of the main issues with utilizing feature sets to identify the afflicted area of spinal cord regions in MRI images is the detection of SCD. Spinal cord atrophy is difficult to automatically identify because of changes in white matter, size, and structure. Differentiating between white and gray matter is crucial in determining whether spinal cord atrophy is detected and how severe it is. SCD repair is still an unsolved medical issue on a global scale. The central nervous system is severely damaged by SCD, and it is extremely difficult to reverse. The complicated underlying pathophysiological obstacles involving the post-traumatic formation of cystic cavities and establishment of thick astrocyte scar have made long-distance axon regeneration a serious issue in neuroscience [9–11]. Spinal cord disorder diagnosis is a complex multi-attribute decision-making (MADM) problem because it involves evaluating multiple interdependent factors, which often contain inherent uncertainties and vagueness. These uncertainties arise from imprecise symptom descriptions, variability in diagnostic criteria, and limitations in medical tests. Theoretical frameworks, such as fuzzy set (FS) [12], soft set (SS) [13], fuzzy soft set (FSS) [14], hypersoft set (HSS) [15, 16] and fuzzy hypersoft set (FHSS) [17], have been developed to handle vagueness, uncertainty, and imprecision in complex information. These methods enable more accurate modeling and decision-making in uncertain environments. The FHS is more adaptable than FS, SS, FSS, and HSS because it extends their capabilities by accommodating multi-attribute and sub-attribute structures within its framework. This adaptability allows it to model complex decision-making scenarios with greater precision by handling higher levels of uncertainty, vagueness, and granularity in data. It combines the strengths of its predecessors while offering enhanced flexibility for managing intricate relationships among attributes, making it highly suitable for real-world problems involving multidimensional and imprecise information. Its adaptability makes it an ideal tool for addressing the uncertainties inherent in medical diagnostic challenges. Its is worth capable to model the complex relationships and variability often present in medical information. As a result, it has been widely employed in medical diagnostics to provide accurate, flexible, and robust decision-making frameworks that improve the reliability of diagnoses and treatment planning in uncertain and dynamic healthcare environments [18–20]. The researchers Xiao [21, 22] and Yang et al. [23] discussed how FS-based strategies could be applied to quantitatively model uncertainty and reduce the uncertainty stemming from subjective human cognition, thereby enhancing decision-making. The growing availability of information through computer networks has made intelligent information processing an emotive topic in information science and applied research. Over the past 30 years, there has been a growing demand for knowledge discovery techniques and information analysis tools, including rule extraction and machine learning. As a result, a variety of information discovery strategies have been developed. An essential mathematical tool for handling vague, inconsistent, and insufficient knowledge is rough set (RS), which was put forth by Pawlak [24–26]. The disciplines of medical diagnosis, pattern recognition, artificial intelligence, data mining, and so on have all effectively used these sets as a helpful tool for handling imprecise information and ambiguity, as noted by Pawlak and Skowron [27]. The RS is regarded as the set of components that either definitely or perhaps belong to the set, and they are related with two crisp sets known as the upper and lower approximations. The equivalence relation is the foundation of Pawlak's RS. This produces an indiscernibility relation, which serves as the mathematical foundation for RS. The hybrid set contexts [28–33] of RST with FS have already been described in literature in order to deal with uncertainties, imperfectness, and indeterminacies.

Relevant literature and motivation

Since FHS has been proven as versatile and adaptable theoretical framework as compared to FSS for quantifying uncertainties and vagueness, therefore, it has been integrated with RS to manage incompleteness as well in literature [34–36]. FSS along with RS plays a significant role in medical diagnosis as it provides a robust framework for analyzing medical data by capturing the imprecision of symptoms, variability in patient responses, and the partial nature of diagnostic knowledge. This approach enables the classification and approximation of medical conditions, aiding in the accurate identification of diseases and improving decision-making in complex and uncertain diagnostic scenarios. Despite being an excellent framework for handling uncertainty and imprecision, the fuzzy soft rough set has not been fully utilized in medical diagnosis. This limited adoption may be due to a lack of awareness or complexity in its application. However, the contributions of scholars [37–41] stand out as remarkable and commendable, showcasing the potential of this approach in addressing complex diagnostic challenges and setting a foundation for future research in this promising area. Similarly, in the case of SCD, no substantial effort has been made to control the associated uncertainty. However, some scholars [42–44] have definitely contributed in this regard. After going through the available literature on SCD, FSS, FHS and RS, it can easily be concluded that the following challenges demand the development of novel theoretical framework:

1. How can unclear and insufficient information related to SCD in particular and medical diagnoses, in general, be handled?
2. In the context of SCD, how can vague or overlapping attribute or their respective values, which are common in complex decision-making scenarios, be captured?
3. How can decision-making be supported in systems with multiple interdependent attributes that may change over time?
4. How can their boundary regions be classified effectively in the context of data that do not fit neatly into predefined categories?

This research introduces the hypersoft fuzzy rough set (HFRS), a novel theoretical framework designed to handle ambiguous, vague, and incomplete information more effectively than existing models. HFRS combines the strengths of fuzzy logic, rough sets, and hypersoft structures, providing a highly adaptable and generalized approach. In decision-making scenarios, particularly for complex problems like spinal cord disorders, HFRS offers targeted solutions to key challenges. Its fuzzy settings adeptly manage ambiguity, while hypersoft arrangements address uncertainty and complexity by considering multiple parameterized factors. Furthermore, the frameworks modified lower and upper approximations enhance the accuracy and precision of modeling incomplete or inconsistent information, making it a robust tool for tackling multifaceted decision-making problems. So in the context of decision-making scenarios, it can effectively handle each of the aforementioned challenges.

1. The main goal is to develop and implement a novel diagnostic framework that combines approximations based on HFRS and MADM. This framework seeks to effectively manage the complexity and uncertainty associated with SCD diagnosis. We propose the incorporation of multiple diagnostic criteria into a comprehensive FHS to increase the accuracy and reliability of SCD diagnoses. This will make it possible to distinguish between various degrees of damage severity more successfully.
2. The approximation space of FHS is developed to describe the pertinent lower and upper approximations to address vagueness and incomplete information together. To pique interest in the context, the suggested concept is illustrated with easily understood numerical examples. The concepts of roughness in the FHS environment have also been explained using the terms roughly fuzzy hypersoft definable, internally fuzzy hypersoft indefinable, externally fuzzy hypersoft indefinable, totally fuzzy hypersoft indefinable, fully FHS, and partition FHS. Additionally, some classical results have been modified in the context of FHS and fuzzy hypersoft approximation space.
3. An intelligent, decision-making system with a strong algorithm has been developed based on the theoretical portion of the current study. The purpose of the suggested algorithm is to quantify the inherent ambiguities, vagueness, and incompleteness to help decision-makers diagnose SCD in patients.The criteria (attributes and their corresponding sub-attributes) are estimated using the weights assigned to them, and the opinions of decision makers are gathered in the form of multiple arguments.

Four sections make up the remaining portion of the paper. Some helpful, necessary definitions are provided in the following section ("Fundamental knowledge" section) to help readers comprehend the key findings. The approximation space of FHS and the associated lower and upper approximations are covered in "Rough approximations of FHS" section. Other pertinent ideas are also presented in this section. An algorithm to assess SCD patients using lower and upper approximations of FHS is developed in "Modified MADM model using FHS rough approximations" section, which is followed by a decisive system. The last section concludes the paper with future directions and possible limitations.

Fundamental knowledge

This section is essential for providing the foundational concepts, definitions, and notations necessary to understand the work. It establishes a common ground for readers by summarizing relevant background information and existing results, ensuring clarity and consistency throughout the paper. This section also situates the research within its broader context and makes the article more accessible and self-contained, allowing readers to follow the new contributions without extensive reliance on external references.

Definition 1 Pawlak [24, 25] Let $\hat{\varrho}$ and $M \neq \phi$ be an indiscernibility relation (equivalence relation) and a finite set respectively, then $(M, \hat{\varrho})$ is called a Pawlak approximation space (information system). If $\hat{\mathbb{X}} \subseteq M$, it is possible that $\hat{\mathbb{X}}$ can be expressed as a union of certain equivalence classes of M, but it is also possible that it cannot. If $\hat{\mathbb{X}}$ can be expressed as the union of certain equivalence classes, then we classify $\hat{\mathbb{X}}$ as definable; otherwise, we deem it as not defined. If $\hat{\mathbb{X}}$ is not possible to be precisely defined, we can estimate it by dividing it into two subsets that can be precisely defined. These subsets are referred to as the lower and upper approximations of $\hat{\mathbb{X}}$.

$$\underline{\hat{\Re}_{\hat{\mathbb{X}}}} = \bigcup \left\{ [\hat{\chi}]_{\hat{\Re}} : [\hat{\chi}]_{\hat{\Re}} \subseteq \hat{\mathbb{X}} \right\}, \tag{1}$$

$$\overline{\hat{\Re}_{\hat{\mathbb{X}}}} = \bigcup \left\{ [\hat{\chi}]_{\hat{\Re}} : [\hat{\chi}]_{\hat{\Re}} \cap \hat{\mathbb{X}} \neq \phi \right\}. \tag{2}$$

Where $\hat{\chi}$ represents an element in M and the equivalence class $[\hat{\chi}]_{\hat{\Re}}$ that it belongs to as a result of the equivalence relation $\hat{\Re}$.

A RS is defined as a pair $\left(\underline{\hat{\Re}_{\hat{\mathbb{X}}}}, \overline{\hat{\Re}_{\hat{\mathbb{X}}}}\right)$. Boundary area refers to the set $\underline{\hat{\Re}_{\hat{\mathbb{X}}}} - \overline{\hat{\Re}_{\hat{\mathbb{X}}}}$. Obviously, if $\underline{\hat{\Re}_{\hat{\mathbb{X}}}} = \overline{\hat{\Re}_{\hat{\mathbb{X}}}}$, then $\hat{\mathbb{X}}$ is definable.

Definition 2 Zadeh [12] Let $M \neq \phi$. A FS $\hat{\mathbb{A}}_{\mathbb{F}}$ in M,

$$\hat{\mathbb{A}}_{\mathbb{F}} = \left\{ \left(\hat{\chi}, \psi(\hat{\chi})\right) : \psi(\hat{\chi}) \in I = [0,1];\ \hat{\chi} \in M \right\} \tag{3}$$

where $\psi(\hat{\chi}) : M \longrightarrow I$ is the fuzzy set's membership function. The collection is symbolized by $\mathcal{C}\left(\hat{\mathbb{A}}_{\mathbb{F}}\right)$.

Definition 3 Molodtsov [13] A SS $\hat{\mathbb{O}}_{\mathbb{S}}$ is a parameterized family of all $\hat{\chi}$ in M. In mathematics, a set made up of entities $\left(\hat{\omega}, \hat{\Theta}_{\hat{\mathbb{O}}}\right)(\hat{\omega})$ is SS where $\hat{\Theta}$ is approximate function from parameters set $\hat{\lambda}$ to the power set of M, $\hat{\Theta} : \hat{\lambda} \longrightarrow \mathrm{P}_M$. The approximate entity of $\hat{\mathbb{O}}_{\mathbb{S}}$ is defined as $\left(\hat{\Theta}_{\hat{\mathbb{O}}}\right)(\hat{\omega})$.

Example 1 Let $M = \{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_3, \hat{\zeta}_4, \hat{\zeta}_5\}$ and $\hat{\mathbb{Z}} = \{\hat{b}_1, \hat{b}_2, \hat{b}_3, \hat{b}_4\}$. Let $(\hat{\Theta}, \hat{\mathbb{Z}})$ be a SS over M which, as indicated below

$$\hat{\Theta}(b_1) = \{\hat{\zeta}_1/1, \hat{\zeta}_2/1, \hat{\zeta}_3/0, \hat{\zeta}_4/0, \hat{\zeta}_5/0\},\ \hat{\Theta}(b_2) = \{\hat{\zeta}_1/1, \hat{\zeta}_2/0, \hat{\zeta}_3/0, \hat{\zeta}_4/0, \hat{\zeta}_5/1\},$$

$$\hat{\Theta}(b_3) = \{\hat{\zeta}_1/0, \hat{\zeta}_2/0, \hat{\zeta}_3/1, \hat{\zeta}_4/1, \hat{\zeta}_5/0\},\ \hat{\Theta}(b_4) = \{\hat{\zeta}_1/0, \hat{\zeta}_2/1, \hat{\zeta}_3/0, \hat{\zeta}_4/1, \hat{\zeta}_5/0\}.$$

Table 1 thus represents the SS $(\hat{\Theta}, \hat{\mathbb{Z}})$.

Definition 4 Smarandache [15] Let $\hat{\mathbb{Z}} = \left\{\hat{b}_1, \hat{b}_2, \hat{b}_3, \hat{b}_4, ..., \hat{b}_{\breve{n}}\right\}$ and $2^{\hat{\Xi}}$ be a set of attributes and a collection of all subsets of $\hat{\Xi}$, respectively. Let the sub values of every attribute $\hat{b}_i, i = 1, 2, 3, ..., \breve{n}$ are contained in disjoint sets $\hat{\Omega}_1$, $\hat{\Omega}_2$, $\hat{\Omega}_3$, ..., $\hat{\Omega}_{\breve{n}}$ respectively. A HSS $(\hat{\Upsilon}, \hat{\Omega})$ can be stated as

Table 1 Tabular representation of SS $(\hat{\Theta}, \hat{\mathbb{Z}})$

	$\hat{\zeta}_1$	$\hat{\zeta}_2$	$\hat{\zeta}_3$	$\hat{\zeta}_4$	$\hat{\zeta}_5$
$\hat{b}_1$	1	1	0	0	0
$\hat{b}_1$	1	0	0	0	1
$\hat{b}_1$	0	0	1	1	0
$\hat{b}_1$	0	1	0	1	0

$$(\hat{\Upsilon}, \hat{\Omega}) = \{(\hat{\nu}_i, \hat{\Upsilon}(\hat{\nu}_i)) : \hat{\Upsilon}(\hat{\nu}_i) \subseteq \hat{\Xi} \wedge \hat{\nu}_i \in \hat{\Omega}\} \tag{4}$$

where $\hat{\Upsilon} : \hat{\Omega} \rightarrow 2^{\hat{\Xi}}$ is an approximated mapping and $\hat{\Omega} = \hat{\Omega}_1 \times \hat{\Omega}_2 \times \hat{\Omega}_3 \times ... \times \hat{\Omega}_{\breve{n}}$ with $\hat{\nu}_i \in \hat{\Omega}$, a $\breve{n}$-argument tuple.

Rough approximations of FHS

Definition 5 Let the sub values of every attribute $\hat{b}_i, i = 1, 2, 3, ..., \breve{n}$ and a set of attributes $\hat{\mathbb{Z}} = \left\{\hat{b}_1, \hat{b}_2, \hat{b}_3, ..., \hat{b}_{\breve{n}}\right\}$ are contained in disjoint sets $\hat{\Omega}_1$, $\hat{\Omega}_2$, $\hat{\Omega}_3$, ..., $\hat{\Omega}_{\breve{n}}$ respectively such that $\hat{\Omega} = \hat{\Omega}_1 \times \hat{\Omega}_2 \times \hat{\Omega}_3 \times ... \times \hat{\Omega}_{\breve{n}}$ with $\hat{\nu}_i \in \hat{\Omega}$, a $\breve{n}$–argument tuple. Let $\hat{\mathbb{S}} = (\hat{\Upsilon}, \hat{\Omega})$ be a FHS and $\hat{\mathbb{G}} = (\hat{\Xi}, \hat{\mathbb{S}})$ be a fuzzy hypersoft approximation space. Based on $\hat{\mathbb{G}}$, the operations listed below can be expressed:

$$\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} = \left\{ \hat{\zeta} \in \hat{\Xi} : \exists\, \hat{\nu} \in \hat{\Omega}, \left[\hat{\zeta} \in \hat{\Upsilon}(\hat{\nu}) \subseteq \hat{M} \right] \right\} \tag{5}$$

$$\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} = \left\{ \hat{\zeta} \in \hat{\Xi} : \exists\, \hat{\nu} \in \hat{\Omega}, \left[\hat{\zeta} \in \hat{\Upsilon}(\hat{\nu}) \cap \hat{M} \neq \phi \right] \right\} \tag{6}$$

that allocate each $\hat{M} \subseteq \hat{\Xi}$ the sets $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$ and $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$ called fuzzy hypersoft $\hat{\mathbb{G}}$-lower and fuzzy hypersoft $\hat{\mathbb{G}}$-upper approximations of $\hat{M}$, respectively. Moreover, the set $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$ is described as fuzzy hypersoft $\hat{\mathbb{G}}$-positive region $\left[\boxplus_{\hat{\mathbb{G}}}(\hat{M})\right]$ of $\hat{M}$, the set $\hat{\Xi} \setminus \overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$ is described as fuzzy hypersoft $\hat{\mathbb{G}}$-negative region $[\boxminus_{\hat{\mathbb{G}}}(\hat{M})]$ of $\hat{M}$, and the set $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} \setminus \overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$ is described as fuzzy hypersoft $\hat{\mathbb{G}}$-boundary region $[\boxtimes_{\hat{\mathbb{G}}}(\hat{M})]$ of $\hat{M}$. If $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} \setminus \overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} \neq \phi$ then $\hat{M}$ is called fuzzy hypersoft $\hat{\mathbb{G}}$-rough set.

In light of Definition 5, it follows that $\hat{M} \subseteq \hat{\Xi}$ is said to be fuzzy hypersoft definable if $[\boxtimes_{\hat{\mathbb{G}}}(\hat{M})] = \phi$ or $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} = \overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$. Furthermore, it's easy to draw the conclusion that $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} \subseteq \hat{M}$ and $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} \subseteq \overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$ $\forall$ $\hat{M} \subseteq \hat{\Xi}$. Regarding the tabular display of FHS $\hat{\mathbb{G}} = (\hat{\Xi}, \hat{\mathbb{S}})$ and its approximations, for every multi-argument tuple $\hat{\nu}$, the following function can be selected.

$$\hat{\nu}(\hat{\zeta}) = \begin{cases} 1 & \hat{\zeta} \in \hat{\Upsilon}(\hat{\nu}) \\ 0 & \hat{\zeta} \notin \hat{\Upsilon}(\hat{\nu}) \end{cases}. \tag{7}$$

Example 2 Let $\hat{\Xi} = \left\{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_3, \hat{\zeta}_4, \hat{\zeta}_5, \hat{\zeta}_6, \hat{\zeta}_7, \hat{\zeta}_8\right\}$, and $\hat{\mathbb{Z}} = \left\{\hat{b}_1, \hat{b}_2, \hat{b}_3, \hat{b}_4\right\}$. The mutually exclusive sets consisting of sub values of attributes $\hat{b}_i$ are $\hat{\Omega}_1 = \left\{\hat{b}_{11}, \hat{b}_{12}\right\}$, $\hat{\Omega}_2 = \{\hat{b}_{21}, \hat{b}_{22}\}$, $\hat{\Omega}_3 = \left\{\hat{b}_{31}, \hat{b}_{32}\right\}$ and $\hat{\Omega}_4 = \left\{\hat{b}_{41}, \hat{b}_{42}\right\}$ such that $\hat{\Omega} = \hat{\Omega}_1 \times \hat{\Omega}_2 \times \hat{\Omega}_3 \times \hat{\Omega}_4 = \{\hat{\nu}_1 = \left(\hat{b}_{11}, \hat{b}_{21}, \hat{b}_{31}, \hat{b}_{41}\right), \hat{\nu}_2 = \left(\hat{b}_{11}, \hat{b}_{21}, \hat{b}_{31}, \hat{b}_{42}\right),$ $\hat{\nu}_3 = \left(\hat{b}_{11}, \hat{b}_{21}, \hat{b}_{32}, \hat{b}_{41}\right), \hat{\nu}_4 = \left(\hat{b}_{11}, \hat{b}_{21}, \hat{b}_{32}, \hat{b}_{42}\right), \hat{\nu}_5 = \left(\hat{b}_{11}, \hat{b}_{22}, \hat{b}_{31}, \hat{b}_{41}\right),$ $\hat{\nu}_6 = \left(\hat{b}_{11}, \hat{b}_{22}, \hat{b}_{31}, \hat{b}_{42}\right), \hat{\nu}_7 = \left(\hat{b}_{11}, \hat{b}_{22}, \hat{b}_{32}, \hat{b}_{41}\right), \hat{\nu}_8 = \left(\hat{b}_{11}, \hat{b}_{22}, \hat{b}_{32}, \hat{b}_{42}\right),$ $\hat{\nu}_9 = \left(\hat{b}_{12}, \hat{b}_{21}, \hat{b}_{31}, \hat{b}_{41}\right), \hat{\nu}_{10} = \left(\hat{b}_{12}, \hat{b}_{21}, \hat{b}_{31}, \hat{b}_{42}\right), \hat{\nu}_{11} = \left(\hat{b}_{12}, \hat{b}_{21}, \hat{b}_{32}, \hat{b}_{41}\right),$ $\hat{\nu}_{12} = \left(\hat{b}_{12}, \hat{b}_{21}, \hat{b}_{32}, \hat{b}_{42}\right), \hat{\nu}_{13} = \left(\hat{b}_{12}, \hat{b}_{22}, \hat{b}_{31}, \hat{b}_{41}\right), \hat{\nu}_{14} = \left(\hat{b}_{12}, \hat{b}_{22}, \hat{b}_{31}, \hat{b}_{42}\right),$ $\hat{\nu}_{15} = \left(\hat{b}_{12}, \hat{b}_{22}, \hat{b}_{32}, \hat{b}_{41}\right), \hat{\nu}_{16} = \left(\hat{b}_{12}, \hat{b}_{22}, \hat{b}_{32}, \hat{b}_{42}\right)\}$. Let $\hat{\mathbb{V}} = \{\hat{\nu}_1, \hat{\nu}_3, \hat{\nu}_5, \hat{\nu}_6, \hat{\nu}_9, \hat{\nu}_{11}, \hat{\nu}_{14}\} \subseteq \hat{\Omega}$ then the respective multi-argument approximate elements are $\hat{\Upsilon}(\hat{\nu_1}) = \{\hat{\zeta}_1, \hat{\zeta}_3, \hat{\zeta}_5, \hat{\zeta}_7, \hat{\zeta}_8\}$, $\hat{\Upsilon}(\hat{\nu_3}) = \{\hat{\zeta}_2, \hat{\zeta}_4, \hat{\zeta}_6\}$, $\hat{\Upsilon}(\hat{\nu_5}) = \{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_5, \hat{\zeta}_6\}$, $\hat{\Upsilon}(\hat{\nu_6}) = \{\hat{\zeta}_1, \hat{\zeta}_4, \hat{\zeta}_7, \hat{\zeta}_8\}$, $\hat{\Upsilon}(\hat{\nu_9}) = \{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_4, \hat{\zeta}_5, \hat{\zeta}_7, \hat{\zeta}_8\}$, $\hat{\Upsilon}(\hat{\nu_{11}}) = \{\hat{\zeta}_1, \hat{\zeta}_3, \hat{\zeta}_4, \hat{\zeta}_7, \hat{\zeta}_8\}$, and $\hat{\Upsilon}(\hat{\nu_{14}}) = \{\hat{\zeta_3}, \hat{\zeta_4}, \hat{\zeta_5}, \hat{\zeta_6}, \hat{\zeta_8}\}$. Now FHS $\hat{\mathbb{S}} = (\hat{\Upsilon}, \hat{\mathbb{V}})$ over $\hat{\Xi}$ is provided using Eq. 7 in Table 2, and fuzzy hypersoft approximation space $\hat{\mathbb{G}} = (\hat{\Xi}, \hat{\mathbb{S}})$. Consider $\hat{M} = \{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_5, \hat{\zeta}_6\} \subseteq \hat{\Xi}$. According to Table 3, $\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_5, \hat{\zeta}_6 \in \hat{\Upsilon}(\hat{\nu_5})$ and $\hat{\Upsilon}(\hat{\nu}_5) \subseteq \hat{M}$, therefore, $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) = \{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_5, \hat{\zeta}_6\}$. Similarly, according to Table 4, the $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) = \{\hat{\zeta}_1, \hat{x}_2, \hat{\zeta}_3, \hat{\zeta}_4, \hat{\zeta}_5, \hat{\zeta}_6, \hat{\zeta}_7, \hat{\zeta}_8\}$. Thus, $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) \neq \overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M})$ and $\hat{M}$ is fuzzy hypersoft $\hat{\mathbb{G}}$-rough set. Similarly, fuzzy hypersoft $\hat{\mathbb{G}}$-positive region of $\hat{M}$ is $[\boxplus_{\hat{\mathbb{G}}}(\hat{M})] = \{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_5, \hat{\zeta}_6\}$, fuzzy hypersoft $\hat{\mathbb{G}}$-negative region of $\hat{M}$ is $[\boxminus_{\hat{\mathbb{G}}}(\hat{M})] = \{\hat{\zeta}_3, \hat{\zeta}_4, \hat{\zeta}_7, \hat{\zeta}_8\}$ and fuzzy hypersoft $\hat{\mathbb{G}}$-boundary of $\hat{M}$ is $[\boxtimes_{\hat{\mathbb{G}}}(\hat{M})] = \{\hat{\zeta}_3, \hat{\zeta}_4, \hat{\zeta}_7, \hat{\zeta}_8\}$.

Definition 6 Let $\hat{\Xi}$, $\hat{\mathbb{S}} = (\hat{\Upsilon}, \hat{\Omega})$ and $\hat{\mathbb{G}} = (\hat{\Xi}, \hat{\mathbb{S}})$ be an initial universe of discourse, a FHS over $\hat{\Xi}$ and fuzzy hypersoft approximation space respectively, then the set $\hat{M} \subseteq \hat{\Xi}$ is called a roughly fuzzy hypersoft definable if $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) \neq \phi$ and $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) \neq \hat{\Xi}$.

Example 3 In Example 2, if we take $\hat{\Upsilon}(\hat{\nu}_1) = \{\hat{\zeta}_3, \hat{\zeta}_7, \hat{\zeta}_8\}$, $\hat{\Upsilon}(\hat{\nu}_{11}) = \{\hat{\zeta}_3, \hat{\zeta}_4, \hat{\zeta}_7, \hat{\zeta}_8\}$ and $\hat{\Upsilon}(\hat{\nu}_{14}) = \{\hat{\zeta}_3, \hat{\zeta}_4, \hat{\zeta}_8\}$ then we have $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) \neq \phi$ and $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) \neq \hat{\Xi}$. Thus, in this case, $\hat{M} \subseteq \hat{\Xi}$ is called a roughly fuzzy hypersoft definable.

The lower and upper approximations of roughly fuzzy hypersoft definable are presented in Table 5 and Table 6, respectively.

Definition 7 Let $\hat{\Xi}$, $\hat{\mathbb{S}} = (\hat{\Upsilon}, \hat{\Omega})$ and $\hat{\mathbb{G}} = (\hat{\Xi}, \hat{\mathbb{S}})$ be an initial universe of discourse, a FHS over $\hat{\Xi}$ and fuzzy hypersoft approximation space respectively, then the set $\hat{M} \subseteq \hat{\Xi}$ is called internally fuzzy hypersoft indefinable if $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) = \phi$ and $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) \neq \hat{\Xi}$.

Example 4 In Example 3, if we take $\hat{\Upsilon}(\hat{\nu}_5) = \{\hat{\zeta}_2, \hat{\zeta}_4, \hat{\zeta}_6, \hat{\zeta}_8\}$, then we have $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) = \phi$ and $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) \neq \hat{\Xi}$. Thus, in this case, $\hat{M} \subseteq \hat{\Xi}$ is called internally fuzzy hypersoft indefinable.

The lower and upper approximations of internally fuzzy hypersoft indefinable are presented in Table 7 and Table 8, respectively.

Definition 8 Let $\hat{\Xi}$, $\hat{\mathbb{S}} = (\hat{\Upsilon}, \hat{\Omega})$ and $\hat{\mathbb{G}} = (\hat{\Xi}, \hat{\mathbb{S}})$ be an initial universe of discourse, a FHS over $\hat{\Xi}$ and fuzzy hypersoft approximation space respectively, then the set $\hat{M} \subseteq \hat{\Xi}$ is called a externally fuzzy hypersoft indefinable if $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) \neq \phi$ and $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) = \hat{\Xi}$.

Example 5 In Example 2, $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) \neq \phi$ and $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) = \hat{\Xi}$. Thus, in this case, $\hat{M} \subseteq \hat{\Xi}$ is called internally fuzzy hypersoft indefinable.

Definition 9 Let $\hat{\Xi}$, $\hat{\mathbb{S}} = (\hat{\Upsilon}, \hat{\Omega})$ and $\hat{\mathbb{G}} = (\hat{\Xi}, \hat{\mathbb{S}})$ be an initial universe of discourse, a FHS over $\hat{\Xi}$ and fuzzy hypersoft approximation space respectively, then the set $\hat{M} \subseteq \hat{\Xi}$ is called totally fuzzy hypersoft indefinable if $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) = \phi$ and $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) = \hat{\Xi}$.

Example 6 In Example 2, if we take $\hat{\Upsilon}(\hat{\nu}_5) = \{\hat{\zeta}_2, \hat{\zeta}_4, \hat{\zeta}_6, \hat{\zeta}_8\}$, then we have $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) = \phi$ and $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}}(\hat{M}) = \hat{\Xi}$. Thus, in this case, $\hat{M} \subseteq \hat{\Xi}$ is called totally fuzzy hypersoft indefinable.

Table 2 Tabular form of FHS $\hat{\mathbb{S}}$

$\hat{\mathbb{S}}$	$\hat{\zeta}_1$	$\hat{\zeta}_2$	$\hat{\zeta}_3$	$\hat{\zeta}_4$	$\hat{\zeta}_5$	$\hat{\zeta}_6$	$\hat{\zeta}_7$	$\hat{\zeta}_8$
$\hat{\nu}_1$	0.9	0	0.4	0	0.5	0	0.6	0.7
$\hat{\nu}_3$	0	0.3	0	0.5	0	0.2	0	0
$\hat{\nu}_5$	0.4	0.5	0	0	0.7	0.8	0	0
$\hat{\nu}_6$	0.3	0	0	0.8	0	0	0.4	0.5
$\hat{\nu}_9$	0.3	0.1	0	0.6	0.5	0	0.8	0.9
$\hat{\nu}_{11}$	0.2	0	0.5	0.8	0	0	0.4	0.3
$\hat{\nu}_{14}$	0	0	0.7	0.5	0.8	0.3	0	0.2

Table 3 Technique for figuring out the lower approximation

$\hat{\zeta}_i$	$\hat{\Upsilon}(\hat{v}_j)$	$\hat{\Upsilon}(\hat{v}_j) \subseteq$ **or** $\nsubseteq \hat{M}$	$\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$
$\hat{\zeta}_1$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Upsilon}(\hat{v}_5) \subseteq \hat{M}$	partially yes
$\hat{\zeta}_2$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_9)$	$\hat{\Upsilon}(\hat{v}_5) \subseteq \hat{M}$	partially yes
$\hat{\zeta}_3$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_1) \nsubseteq \hat{M}$	No
$\hat{\zeta}_4$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14}) \nsubseteq \hat{M}$	No
$\hat{\zeta}_5$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_5) \subseteq \hat{M}$	partially yes
$\hat{\zeta}_6$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_5) \subseteq \hat{M}$	partially yes
$\hat{\zeta}_7$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Upsilon}(\hat{v}_6) \nsubseteq \hat{M}$	No
$\hat{\zeta}_8$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14}) \nsubseteq \hat{M}$	No

Table 4 Technique for figuring out the upper approximation

$\hat{\zeta}_i$	$\hat{\Upsilon}(\hat{v}_j)$	$\hat{\Upsilon}(\hat{v}_j) \cap \hat{M} \neq \phi$ **or** $= \phi$	$\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$
$\hat{\zeta}_1$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Upsilon}(\hat{v}_5) \cap \hat{M} \neq \phi$	partially yes
$\hat{\zeta}_2$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_9)$	$\hat{\Upsilon}(\hat{v}_3) \cap \hat{M} \neq \phi$	partially yes
$\hat{\zeta}_3$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_1) \cap \hat{M} \neq \phi$	partially Yes
$\hat{\zeta}_4$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14}) \cap \hat{M} \neq \phi$	partially Yes
$\hat{\zeta}_5$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_9) \cap \hat{M} \neq \phi$	partially Yes
$\hat{\zeta}_6$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_5) \cap \hat{M} \neq \phi$	partially Yes
$\hat{\zeta}_7$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Upsilon}(\hat{v}_6) \cap \hat{M} \neq \phi$	partially Yes
$\hat{\zeta}_8$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14}) \cap \hat{M} \neq \phi$	partially Yes

Table 5 Lower approximation for roughly fuzzy hypersoft definable

$\hat{\zeta}_i$	$\hat{\Upsilon}(\hat{v}_j)$	$\hat{\Upsilon}(\hat{v}_j) \subseteq$ **or** $\nsubseteq \hat{M}$	$\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$
$\hat{\zeta}_1$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Psi}(\hat{v}_5) \subseteq \hat{M}$	partially Yes
$\hat{\zeta}_2$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_9)$	$\hat{\Upsilon}(\hat{v}_3) \subseteq \hat{M}$	partially Yes
$\hat{\zeta}_3$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_1) \nsubseteq \hat{M}$	No
$\hat{\zeta}_4$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14}) \nsubseteq \hat{M}$	No
$\hat{\zeta}_5$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_5) \subseteq \hat{M}$	partially Yes
$\hat{\zeta}_6$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_5) \subseteq \hat{M}$	partially Yes
$\hat{\zeta}_7$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Upsilon}(\hat{v}_6) \nsubseteq \hat{M}$	No
$\hat{\zeta}_8$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14}) \nsubseteq \hat{M}$	No

Proposition 1 and Theorem 5 (page 906), which were discussed by Feng et al. [45], are the results that follow, updated under the fuzzy hypersoft set context.

Theorem 1 *Let* $\hat{\Xi}$, $\hat{\mathbb{S}} = (\hat{\Upsilon}, \hat{\Omega})$ *and* $\hat{\mathbb{G}} = (\hat{\Xi}, \hat{\mathbb{S}})$ *be an initial universe of discourse, a FHS over* $\hat{\Xi}$ *and fuzzy hypersoft approximation space respectively, then the following versions of fuzzy hypersoft* $\hat{\mathbb{G}}$*-lower approximation* $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$ *and fuzzy hypersoft* $\hat{\mathbb{G}}$*-upper approximations* $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$ *are valid for all* $\hat{M} \subseteq \hat{\Xi}$ $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} = \bigcup_{\hat{v} \in \hat{V}} \left\{ \hat{\Upsilon}(\hat{v}) : \hat{\Upsilon}(\hat{v}) \subseteq \hat{M} \right\}$ *and* $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} = \bigcup_{\hat{v} \in \hat{V}} \left\{ \hat{\Upsilon}(\hat{v}) : \hat{\Upsilon}(\hat{v}) \cap \hat{M} \neq \phi \right\}$.

Table 6 Upper approximation for roughly fuzzy hypersoft definable

$\hat{\zeta}_i$	$\hat{\Upsilon}(\hat{v}_j)$	$\hat{\Upsilon}(\hat{v}_j) \cap \hat{M} \neq \phi$ **or** $= \phi$	$\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$
$\hat{\zeta}_1$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Upsilon}(\hat{v}_5) \cap \hat{M} \neq \phi$	partially Yes
$\hat{\zeta}_2$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_9)$	$\hat{\Upsilon}(\hat{v}_3) \cap \hat{M} \neq \phi$	partially Yes
$\hat{\zeta}_3$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_1) \cap \hat{M} = \phi$	No
$\hat{\zeta}_4$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14}) \cap \hat{M} \neq \phi$	Yes
$\hat{\zeta}_5$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_9) \cap \hat{M} \neq \phi$	partially Yes
$\hat{\zeta}_6$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_5) \cap \hat{M} \neq \phi$	partially Yes
$\hat{\zeta}_7$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Upsilon}(\hat{v}_6) \cap \hat{M} \neq \phi$	partially Yes
$\hat{\zeta}_8$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14}) \cap \hat{M} \neq \phi$	partially Yes

Proof Definition 5 makes the proof simple to understand. That can also be demonstrated and validated by the example that follows, though.

Example 7 It is evident from Example 2's computations and underlying presumptions that only $\hat{\Upsilon}(\hat{v}_5) \subseteq \hat{M}$, therefore, $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} = \hat{\Upsilon}(\hat{v}_5) = \{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_5, \hat{\zeta}_6\}$. Since, $\hat{\Upsilon}(\hat{v}_1) \cap \hat{M} \neq \phi$, $\hat{\Upsilon}(\hat{v}_3) \cap \hat{M} \neq \phi$, $\hat{\Upsilon}(\hat{v}_5) \cap \hat{M} \neq \phi$, $\hat{\Upsilon}(\hat{v}_6) \cap \hat{M} \neq \phi$, $\hat{\Upsilon}(\hat{v}_9) \cap \hat{M} \neq \phi$, $\hat{\Upsilon}(\hat{v}_{11}) \cap \hat{M} \neq \phi$, and $\hat{\Upsilon}(\hat{v}_{14}) \cap \hat{M} \neq \phi$, therefore, $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})} = \hat{\Upsilon}(\hat{v}_1) \cup \hat{\Upsilon}(\hat{v}_3) \cup \hat{\Upsilon}(\hat{v}_5) \cup \hat{\Upsilon}(\hat{v}_6) \cup \hat{\Upsilon}(\hat{v}_9) \cup \hat{\Upsilon}(\hat{v}_{11}) \cup \hat{\Upsilon}(\hat{v}_{14}) = \{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_3, \hat{\zeta}_4, \hat{\zeta}_5, \hat{\zeta}_6, \hat{\zeta}_7, \hat{\zeta}_8\}$.

Theorem 2 *Let* $\hat{\Xi}$, $\hat{\mathbb{S}} = (\hat{\Upsilon}, \hat{\Omega})$ *and* $\hat{\mathbb{G}} = (\hat{\Xi}, \hat{\mathbb{S}})$ *be an initial universe of discourse, a FHS over* $\hat{\Xi}$ *and a fuzzy hypersoft approximation space respectively, then the following results are valid for all* $\hat{K}, \hat{M} \subseteq \hat{\Xi}$:

Table 7 Lower approximation for internally fuzzy hypersoft indefinable

$\hat{\zeta}_i$	$\hat{\Psi}(\hat{v}_j)$	$\hat{\Upsilon}(\hat{v}_j) \subseteq$ **or** $\nsubseteq \hat{M}$	$\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$
$\hat{\zeta}_1$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Upsilon}(\hat{v}_5) \nsubseteq \hat{M}$	No
$\hat{\zeta}_2$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_9)$	$\hat{\Upsilon}(\hat{v}_3) \nsubseteq \hat{M}$	No
$\hat{\zeta}_3$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_1) \nsubseteq \hat{M}$	No
$\hat{\zeta}_4$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14}) \nsubseteq \hat{M}$	No
$\hat{\zeta}_5$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_5) \nsubseteq \hat{M}$	No
$\hat{\zeta}_6$	$\hat{\Upsilon}(\hat{v}_3), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_5) \nsubseteq \hat{M}$	No
$\hat{\zeta}_7$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Upsilon}(\hat{v}_6) \nsubseteq \hat{M}$	No
$\hat{\zeta}_8$	$\hat{\Upsilon}(\hat{v}_1), \hat{\Upsilon}(\hat{v}_5), \hat{\Upsilon}(\hat{v}_6), \hat{\Upsilon}(\hat{v}_9), \hat{\Upsilon}(\hat{v}_{11}), \hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14}) \nsubseteq \hat{M}$	No

Table 8 Upper approximation for internally fuzzy hypersoft indefinable

$\hat{\zeta}_i$	$\hat{\Upsilon}(\hat{v}_j)$	$\hat{\Upsilon}(\hat{v}_j)\cap\hat{M}\neq\phi$ **or** $=\phi$	$\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$
$\hat{\zeta}_1$	$\hat{\Upsilon}(\hat{v}_1),\hat{\Upsilon}(\hat{v}_5),\hat{\Upsilon}(\hat{v}_6),\hat{\Upsilon}(\hat{v}_9),\hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Upsilon}(\hat{v}_5)\cap\hat{M}\neq\phi$	partially Yes
$\hat{\zeta}_2$	$\hat{\Upsilon}(\hat{v}_3),\hat{\Upsilon}(\hat{v}_5),\hat{\Upsilon}(\hat{v}_9)$	$\hat{\Upsilon}(\hat{v}_3)\cap\hat{M}\neq\phi$	partially Yes
$\hat{\zeta}_3$	$\hat{\Upsilon}(\hat{v}_1),\hat{\Upsilon}(\hat{v}_{11}),\hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_1)\cap\hat{M}=\phi$	No
$\hat{\zeta}_4$	$\hat{\Upsilon}(\hat{v}_3),\hat{\Upsilon}(\hat{v}_6),\hat{\Upsilon}(\hat{v}_9),\hat{\Upsilon}(\hat{v}_{11}),\hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14})\cap\hat{M}\neq\phi$	partially Yes
$\hat{\zeta}_5$	$\hat{\Upsilon}(\hat{v}_1),\hat{\Upsilon}(\hat{v}_5),\hat{\Upsilon}(\hat{v}_9),\hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_9)\cap\hat{M}\neq\phi$	partially Yes
$\hat{\zeta}_6$	$\hat{\Upsilon}(\hat{v}_3),\hat{\Upsilon}(\hat{v}_5),\hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_5)\cap\hat{M}\neq\phi$	partially Yes
$\hat{\zeta}_7$	$\hat{\Upsilon}(\hat{v}_1),\hat{\Upsilon}(\hat{v}_6),\hat{\Upsilon}(\hat{v}_9),\hat{\Upsilon}(\hat{v}_{11})$	$\hat{\Upsilon}(\hat{v}_6)\cap\hat{M}\neq\phi$	partially Yes
$\hat{\zeta}_8$	$\hat{\Upsilon}(\hat{v}_1),\hat{\Upsilon}(\hat{v}_5),\hat{\Upsilon}(\hat{v}_6),\hat{\Upsilon}(\hat{v}_9),\hat{\Upsilon}(\hat{v}_{11}),\hat{\Upsilon}(\hat{v}_{14})$	$\hat{\Upsilon}(\hat{v}_{14})\cap\hat{M}\neq\phi$	partially Yes

1. $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\phi)}=\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\phi)}=\phi$.
2. $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\Xi})}=\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\Xi})}=\bigcup_{\hat{v}\in\hat{V}}\hat{\Upsilon}(\hat{v})$.
3. *If* $\hat{K}\subseteq\hat{M}$ *then* $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{K})}\subseteq\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$.
4. *If* $\hat{K}\subseteq\hat{M}$ *then* $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{K})}\subseteq\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$.
5. $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{K}\cap\hat{M})}\subseteq\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{K})}\cap\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$.
6. $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{K}\cup\hat{M})}\supseteq\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{K})}\cup\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$.
7. $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{K}\cap\hat{M})}\subseteq\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{K})}\cap\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$.
8. $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{K}\cup\hat{M})}=\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{K})}\cup\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{M})}$.

Definition 5, Example 2, and Theorem 1 make it simple to demonstrate the aforementioned results. Furthermore, the work of Feng et al. [45] (Theorem 5, page 906) can be useful in this context.

Definition 10 Let $\hat{\mathbb{S}}=(\hat{\Upsilon},\hat{\Omega})$ be a FHS over $\hat{\Xi}$. If $\bigcup_{\hat{v}\in\hat{D}}\hat{\Upsilon}(\hat{v})=\hat{\Xi}$, then $\hat{\mathbb{S}}$ is known as fully FHS.

Definition 11 Let $\hat{\mathbb{S}}=(\hat{\Upsilon},\hat{\Omega})$ be a FHS over $\hat{\Xi}$. If $\{\hat{\Upsilon}(\hat{v}):\hat{v}\in\hat{V}\}$ forms a partition of $\hat{\Xi}$, then $\hat{\mathbb{S}}$ is known as partition FHS.

Modified MADM model using FHS rough approximations

Feng [46] used soft rough approximations to solve MADM problems. His method increases the accuracy of selecting the optimal object. This study used fuzzy hypersoft approximations based on Feng's approach to detect spinal cord disorder.

Proposed method

Let $\hat{\Xi}=\{\hat{\zeta}_1,\hat{\zeta}_2,...,\hat{\zeta}_{\breve{m}}\}$ and $\hat{\mathbb{Z}}$ be a set of alternatives and a set of evaluating parameters. Let $\hat{\Omega}$ be the Cartesian product of disjoint sets consisting of sub values of parameters. Take $\hat{\mathbb{V}}=\{\hat{v}_1,\hat{v}_2,...,\hat{v}_{\breve{n}}\}\subseteq\hat{\Omega}$. Let $\hat{\mathbb{S}}=(\hat{\Upsilon},\hat{\Omega})$ be a full FHS over $\hat{\Xi}$ with real representation. Let $\hat{\mho}=\{\mathbb{O}_1,\mathbb{O}_2,...,\mathbb{O}_{\breve{p}}\}$ be a set of experts who are responsible to point out "the appropriate alternatives" using their primary evaluation that are represented by $\hat{\mathbb{H}}_i$ with respect to FHS approximation space $\hat{\mathbb{G}}=(\hat{\Xi},\hat{\mathbb{S}})$. Based on $\hat{\mathbb{H}}_i$ for all $\mathbb{O}_i$, an evaluation SS $\hat{\mathbb{S}}_1=(\hat{\Upsilon},\hat{\mho})$ can be constructed where $\hat{\Upsilon}:\hat{\mho}\to 2^{\hat{\Xi}}$ is a mapping given by $\hat{\Upsilon}(\mathbb{O}_i)=\hat{\mathbb{H}}_i$. The FHS lower approximation $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$ and FHS upper approximation $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$ are computed for having certainly and possibly optimum alternatives based of $\hat{\mathbb{H}}_i$. Using these FHS rough approximations, the following SSs over $\hat{\Xi}$ are obtained:

$\overleftarrow{\hat{\mathbb{S}}_1}=(\overleftarrow{\hat{\Upsilon}},\hat{\mho})$ where $\overleftarrow{\hat{\Upsilon}}:\hat{\mho}\to 2^{\hat{\Xi}}$ is an approximate mapping given by $\overleftarrow{\hat{\Upsilon}}(\mathbb{O}_i)=\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$,

$\overrightarrow{\hat{\mathbb{S}}_1}=(\overrightarrow{\hat{\Upsilon}},\hat{\mho})$ where $\overrightarrow{\hat{\Upsilon}}:\hat{\mho}\to 2^{\hat{\Xi}}$ is an approximate mapping given by $\overrightarrow{\hat{\Upsilon}}(\mathbb{O}_i)=\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$.

It is noteworthy that FSs could also be used to indicate the evaluation result for the complete expert set $\hat{\mho}$. Let $\hat{\Delta}_{\hat{M}}$ represents the characteristic function $\hat{M}\subseteq\hat{\Xi}$. Using SS $\hat{\mathbb{S}}_1=(\hat{\Upsilon},\hat{\mho})$, we can construct the following FSs in $\hat{\Xi}$:

$\chi_{\hat{\mathbb{S}}_1}:\hat{\Xi}\to[0,1]$ defined by

$$\zeta_{\hat{\mathbb{S}}_1}(\hat{\zeta}_j)=\left(\frac{1}{\breve{p}}\right)\sum_{i=1}^{\breve{p}}\Delta_{\hat{\Upsilon}(\hat{\mathbb{O}}_i)}\left(\hat{\zeta}_j\right), \tag{8}$$

$\chi_{\overleftarrow{\hat{\mathbb{S}}_1}}:\hat{\Xi}\to[0,1]$ defined by

$$\chi_{\overleftarrow{\hat{\mathbb{S}}_1}}(\hat{\zeta}_j)=\left(\frac{1}{\breve{p}}\right)\sum_{i=1}^{\breve{p}}\Delta_{\overleftarrow{\hat{\Upsilon}}(\hat{\mathbb{O}}_i)}\left(\hat{\zeta}_j\right), \tag{9}$$

$\chi_{\overrightarrow{\hat{\mathbb{S}}_1}}:\hat{\Xi}\to[0,1]$ defined by

$$\zeta_{\overrightarrow{\hat{\mathbb{S}}_1}}(\hat{\zeta}_j)=\left(\frac{1}{\breve{p}}\right)\sum_{i=1}^{\breve{p}}\Delta_{\overrightarrow{\hat{\Upsilon}}(\hat{\mathbb{O}}_i)}\left(\hat{\zeta}_j\right), \tag{10}$$

where $\hat{\Upsilon}(\hat{\mathbb{O}}_i)=\hat{\mathbb{H}}_i$, $\overleftarrow{\hat{\Upsilon}}(\hat{\mathbb{O}}_i)=\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$, $\overrightarrow{\hat{\Upsilon}}(\hat{\mathbb{O}}_i)=\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$ and $j=1,2,...,m$.

Now, based on above FSs, construct fuzzy SS $(\hat{\lambda},\hat{L})$ characterized by mapping $\hat{\lambda}:\hat{F}\to\mathbb{L}^{\hat{\Xi}}$ defined by $\hat{\lambda}(LC)=\chi_{\overrightarrow{\hat{\mathbb{S}}_1}}$, $\hat{\lambda}(MC)=\chi_{\hat{\mathbb{S}}_1}$ and $\hat{\lambda}(HC)=\chi_{\overleftarrow{\hat{\mathbb{S}}_1}}$, where $\mathbb{L}^{\hat{\Xi}}$

is the collection of fuzzy subsets over $\hat{\Xi}$, $\hat{F}$ is the collection of elements categorized as low confidence (LC), medium confidence (MC) and high confidence (HC). Now calculate weighted evaluation values $\hat{\Upsilon}(\hat{\zeta}_j)$ of alternatives $\hat{\zeta}_j$ using the following formula:

$$\hat{\Upsilon}(\hat{\zeta}_j) = \hat{\eta}_{LC} \times \hat{\lambda}(LC)(\hat{\zeta}_j) + \hat{\eta}_{MC} \times \hat{\lambda}(MC)(\hat{\zeta}_j) + \hat{\eta}_{HC} \times \hat{\lambda}(HC)(\hat{\zeta}_j) \tag{11}$$

where $\hat{\eta}_{LC}$, $\hat{\eta}_{MC}$, and $\hat{\eta}_{HC}$ are weights such that $\hat{\eta}_{LC} + \hat{\eta}_{MC} + \hat{\eta}_{HC} = 1$. Finally, the alternative $\hat{\zeta}_j$ with maximum weighted evaluation value $\hat{\Upsilon}(\hat{\zeta}_j)$, is selected.

Based on the above-discussed MADM system, a robust algorithm is proposed as

actions that forward strategic objectives by carefully examining these values. This meticulous process guarantees that choices are transparent, data-driven, and tailored to yield the best outcomes. The criteria (parameters) and sub-criteria (sub parametric values) as shown in Table 9, Figs. 2, 3, 4, and 5 are adopted with partial modification in this study in accordance with the findings of the researchers' investigation [47–51] in the framework of MADM. Similar to this, selecting a collection of alternatives is a crucial first step when dealing with a decision-making challenge since it gives us a variety of options to consider and contrast, promoting in-depth examination. This method makes it easier to find the most workable answer, leading to fair and well-informed decision-making.

Algorithm 1 Decision assisted system based on rough approximation of FHS

1. Input:

1.1. Let $\hat{\Xi}$, $\hat{\mathbb{Z}}$, and $\hat{\mho}$ be a set of alternatives, a set of evaluating parameters and a set of specialists (experts).

1.2. Let $\hat{\mathbb{V}} \subseteq \hat{\Omega}$ such that $\hat{\Omega}$ is the Cartesian product of attribute-valued disjoint sets consisting multi-argument tuples.

1.3. Let $\hat{\mathbb{S}} = (\hat{\Upsilon}, \hat{\Omega})$ be a full FHS over $\hat{\Xi}$ with real representation.

2. Construction:

2.1. Based on primary evaluation $\hat{\mathbb{H}}_i$ for all experts $\mathbb{O}_i$, construct an evaluation SS $\hat{\mathbb{S}}_1 = (\hat{\Upsilon}, \hat{\mho})$ where $\hat{\Upsilon} : \hat{\mho} \to 2^{\hat{\Xi}}$ is a mapping given by $\hat{\Upsilon}(\mathbb{O}_i) = \hat{\mathbb{H}}_i$.

2.2. Construct FHS lower approximation $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$ and FHS upper approximation $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$.

2.3. Construct $\overleftarrow{\hat{\mathbb{S}}_1} = (\overleftarrow{\hat{\Upsilon}}, \hat{\mho})$ where $\overleftarrow{\hat{\Upsilon}} : \hat{\mho} \to 2^{\hat{\Xi}}$ such that $\overleftarrow{\hat{\Upsilon}}(\mathbb{O}_i) = \overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$.

2.4. Construct $\overrightarrow{\hat{\mathbb{S}}_1} = (\overrightarrow{\hat{\Upsilon}}, \hat{\mho})$ where $\overrightarrow{\hat{\Upsilon}} : \hat{\mho} \to 2^{\hat{\Xi}}$ such that $\overrightarrow{\hat{\Upsilon}}(\mathbb{O}_i) = \overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$.

3. Computation:

3.1. Compute FSs $\chi_{\hat{\mathbb{S}}_1}$, $\zeta_{\overleftarrow{\hat{\mathbb{S}}_1}}$ and $\chi_{\overrightarrow{\hat{\mathbb{S}}_1}}$ corresponding to $\hat{\mathbb{S}}_1$, $\overleftarrow{\hat{\mathbb{S}}_1}$ and $\overrightarrow{\hat{\mathbb{S}}_1}$ respectively.

3.2. Compute fuzzy SS $(\hat{\lambda}, \hat{F})$ using FSs $\chi_{\hat{\mathbb{S}}_1}$, $\chi_{\overleftarrow{\hat{\mathbb{S}}_1}}$ and $\chi_{\overrightarrow{\hat{\mathbb{S}}_1}}$.

3.3. Assign weights to each element in $\hat{F}$ and compute weighted values using Equation 8, Equation 9, Equation 10 and Equation 11.

4. Output:

4.1. Based on weighted values, rank the alternatives and select the alternative which secures highest weighted value.

The flowchart of the proposed algorithm is presented in Fig. 1.

Mechanism for obtaining raw data

The parameters and their corresponding sub-parametric values play a crucial role in shaping the decisions and outcomes. Parameters specify the main criteria or elements that influence choices. Sub-parametric values offer a higher degree of specificity, making it possible to evaluate each parameter precisely. Decision-makers can evaluate options, analyze trade-offs, and prioritize

Only three main criteria (attributes) the neurological status criteria, imaging data criteria, and functional abilities criteria have been chosen for the evaluation of SCD patients in Table 9, despite the fact that there are numerous criteria in the literature to diagnose the disease. This is because they have enough subattribute values to meet the demands of the FHS environment.

Hypothetical case study

After falling from a considerable height, a 35-year-old man experienced acute back pain and partial loss of movement

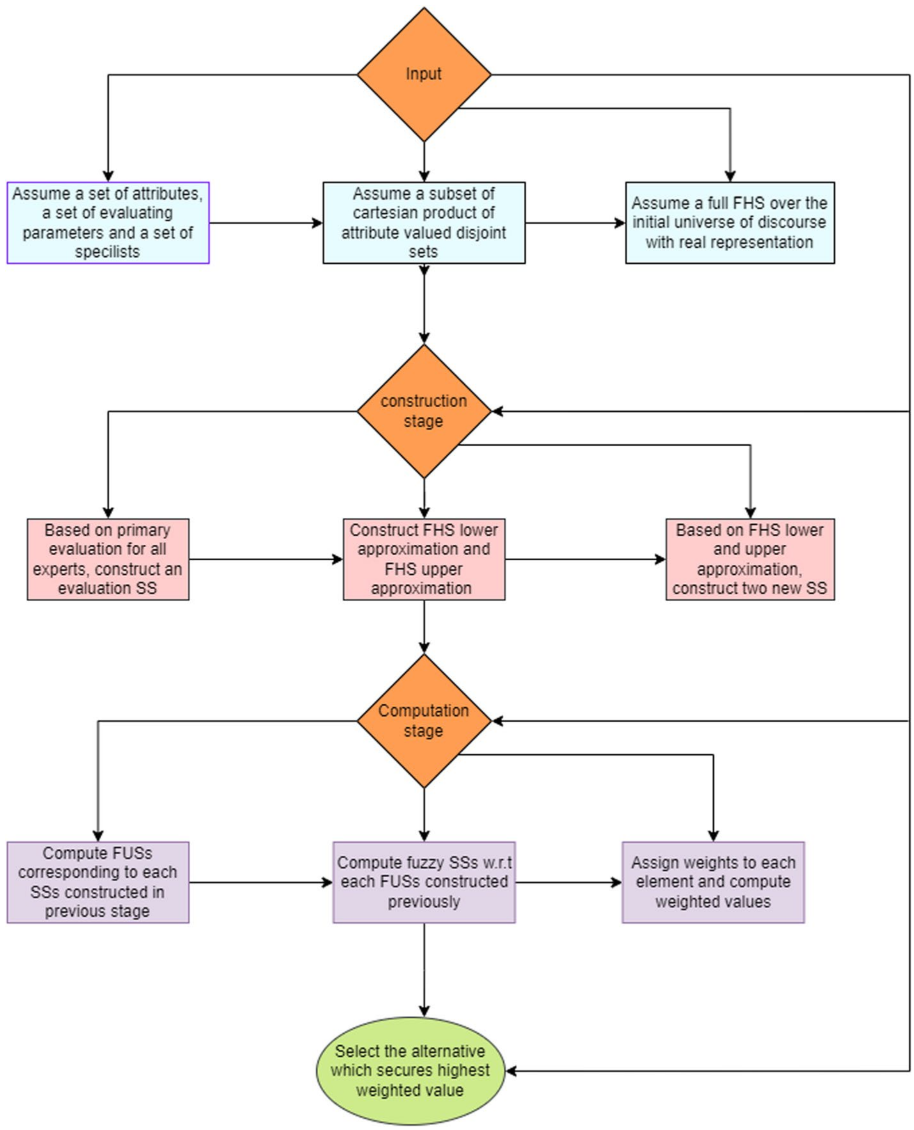

Fig. 1 Flowchart of proposed algorithm

Table 9 Adopted parameters and their respective sub parametric values

Parameters	Sub parametric values
$\hat{b}_1$ = Neurological Status Criteria	$\hat{b}_{11}$ = Motor function, $\hat{b}_{12}$ = Sensory function, $\hat{b}_{13}$ = Reflexe, $\hat{b}_{14}$ = Autonomic function, $\hat{b}_{15}$ = Level of injury
$\hat{b}_2$ = Imaging Data Criteria	$\hat{b}_{21}$ = Magnetic resonance imaging (MRI), $\hat{b}_{22}$ = computed tomography (CT) scan, $\hat{b}_{23}$ = X-ray, $\hat{b}_{24}$ = Diffusion tensor imaging (DTI), $\hat{b}_{25}$ = Ultrasound
$\hat{b}_3$ = Functional Abilities Criteria	$\hat{b}_{31}$ = Activities of daily living (ADL), $\hat{b}_{32}$ = Mobility, $\hat{b}_{33}$ = Upper limb function, $\hat{b}_{34}$ = Respiratory function, $\hat{b}_{35}$ = Quality of life (QoL) assessments

in his lower limbs. He then went to the emergency department. A SCD with diminished motor and sensory function below the T10 vertebral level was shown by the preliminary neurological evaluation. An MRI and CT scan, among other imaging tests, verified a fracture at the T10 vertebra with compression of the spinal cord. Regarding the degree of nerve injury and possible healing prospects, uncertainty remained despite conventional diagnostic techniques. A MADM system that makes use of rough approximations of fuzzy hypersoft set was used to address issue. In order

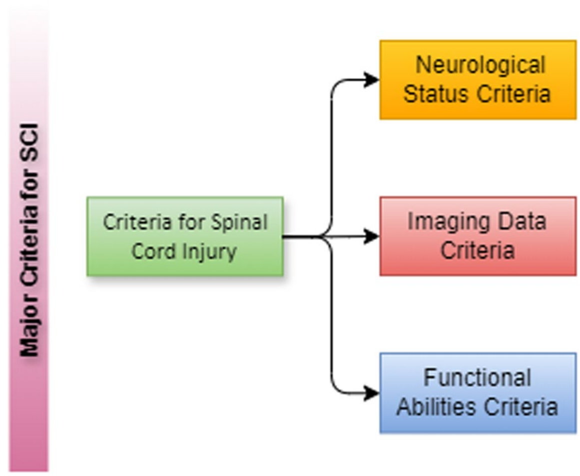

Fig. 2 Major criteria for SCD

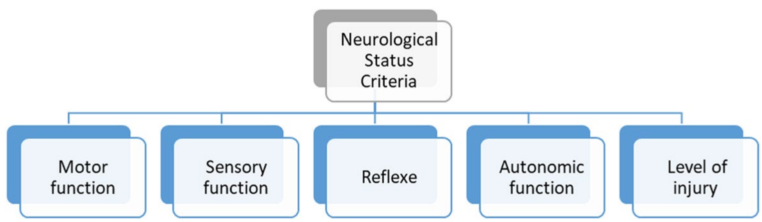

Fig. 3 Sub-criteria of neurological status criteria

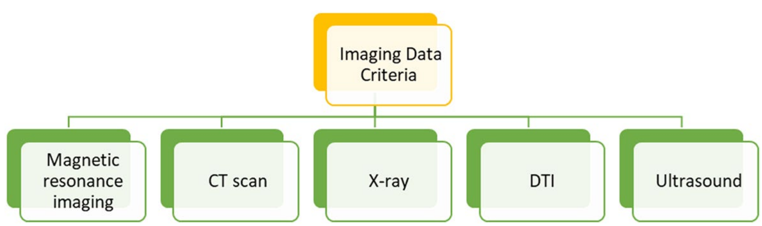

Fig. 4 Sub-criteria of imaging data criteria

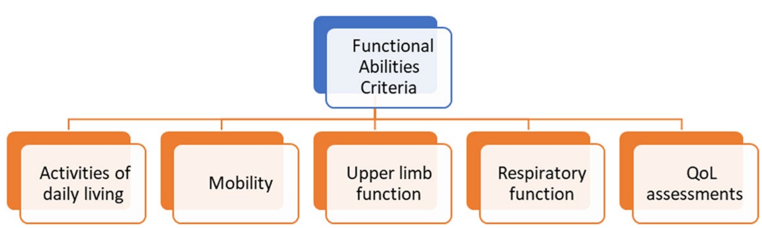

Fig. 5 Sub-criteria of functional abilities criteria

to offer a thorough assessment, this method included neurological status, imaging data, and functional ability criteria. Over a six-month period, the patient's motor function and overall quality of life significantly improved as a result of the customized treatment plan the model recommended, which involved heavy rehabilitation after surgery. The management of complex SCD is made easier by the use of advanced diagnostic frameworks, which are especially useful when dealing with complex and ambiguous clinical data. A committee consisting of three specialists $\hat{\mathbb{O}}_1$, $\hat{\mathbb{O}}_2$, and $\hat{\mathbb{O}}_3$ is formed to guarantee that the study is thorough and knowledgeable. These experts contribute knowledge from a variety of fields related to SCD, including neurology, radiology, and rehabilitation medicine. Their responsibilities include evaluating the available options, offering advice based on their clinical background, and directing the study's decision-making process. The utilization of a collaborative strategy guarantees that the study takes into account various viewpoints and that the selected alternatives undergo thorough evaluation, resulting in conclusions that are more dependable and relevant. As per the committee, potential injuries are $\hat{\xi}_1$ = complete SCD, $\hat{\xi}_2$ = incomplete SCD, $\hat{\xi}_3$ = cervical SCD, $\hat{\xi}_4$ = thoracic SCD, $\hat{\xi}_5$ = lumbar SCD, $\hat{\xi}_6$ = sacral SCD, $\hat{\xi}_7$ = central cord syndrome. These injuries make up a set of parameters $\hat{\Xi} = \{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_3, \hat{\zeta}_4, \hat{\zeta}_5, \hat{\zeta}_6, \hat{\zeta}_7\}$. After experts confer with one another, the parameters selected for assessment are $\hat{b}_1 =$ neurological status criteria, $\hat{b}_2 =$ imaging data criteria, and $\hat{b}_3 =$ functional abilities criteria. These attributes make up the set $\hat{\mathbb{Z}} = \{\hat{b}_1, \hat{b}_2, \hat{b}_3\}$. The sub parametric values of the parameters are included in the sets based on a preferential basis. $\hat{\Omega}_1 = \{\hat{b}_{11}, \hat{b}_{12}\}$, $\hat{\Omega}_2 = \{\hat{b}_{21}, \hat{b}_{22}\}$, and $\hat{\Omega}_3 = \{\hat{b}_{31}, \hat{b}_{32}\}$ respectively. Table 9 presents the sub parametric values in detail. In order to obtain multi-argument tuples, $\hat{\Omega} = \hat{\Omega}_1 \times \hat{\Omega}_2 \times \hat{\Omega}_3$ is calculated with the components $\hat{\nu}_1 = (\hat{b}_{11}, \hat{b}_{21}, \hat{b}_{31})$, $\hat{\nu}_2 = (\hat{b}_{11}, \hat{b}_{21}, \hat{b}_{32})$, $\hat{\nu}_3 = (\hat{b}_{11}, \hat{b}_{22}, \hat{b}_{31})$, $\hat{\nu}_4 = (\hat{b}_{11}, \hat{b}_{22}, \hat{b}_{32})$, $\hat{\nu}_5 = (\hat{b}_{12}, \hat{b}_{21}, \hat{b}_{31})$, $\hat{\nu}_6 = (\hat{b}_{12}, \hat{b}_{21}, \hat{b}_{32})$, $\hat{\nu}_7 = (\hat{b}_{12}, \hat{b}_{22}, \hat{b}_{31})$, and $\hat{\nu}_8 = (\hat{b}_{12}, \hat{b}_{22}, \hat{b}_{32})$. Take $\hat{\mathbb{V}} = \{\hat{\nu}_2, \hat{\nu}_4, \hat{\nu}_6, \hat{\nu}_8\} \subseteq \hat{\Omega}$ for additional assessment, giving precedence to $\hat{b}_{32}$ in $\hat{\Omega}_3$. Using Eq. 7, a full FHS $\hat{\mathbb{S}} = (\hat{\Upsilon}, \hat{\Omega})$ is built over $\hat{\Xi}$ with accurate tabular representation (see Table 10). The FHS $\hat{\mathbb{S}}$ is built utilizing the approximations $\hat{\Upsilon}(\hat{\nu}_2) = \{\hat{\zeta}_5, \hat{\zeta}_7\}$, $\hat{\Upsilon}(\hat{\nu}_4) = \{\hat{\zeta}_1, \hat{\zeta}_4, \hat{\zeta}_6, \hat{\zeta}_7\}$, $\hat{\Upsilon}(\hat{\nu}_6) = \{\hat{\zeta}_1, \hat{\zeta}_3\}$ and $\hat{\Upsilon}(\hat{\nu}_8) = \{\hat{\zeta}_2, \hat{\zeta}_4\}$. Currently, primary assessments $\hat{\mathbf{H}}_i$ from experts are gathered to evaluate spinal cord disorder damage. Based on $\hat{\mathbf{H}}_i$, an evaluation SS $\hat{\mathbb{S}}_1 = (\hat{\Upsilon}, \hat{\mho})$ is constructed with $\hat{\Upsilon} : \hat{\mho} \to 2^{\hat{\Xi}}$ such that $\hat{\Upsilon}(\mathbb{O}_i) = \hat{\mathbb{H}}_i$. Table 11 presents them and demonstrates that $\hat{\mathbf{H}}_1 = \{\hat{\zeta}_4, \hat{\zeta}_5, \hat{\zeta}_7\}$, $\hat{\mathbf{H}}_2 = \{\hat{\zeta}_1, \hat{\zeta}_3, \hat{\zeta}_7\}$, and $\hat{\mathbf{H}}_3 = \{\hat{\zeta}_2, \hat{\zeta}_4, \hat{\zeta}_5\}$.

Now FHS lower approximations $\overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$ and FHS upper approximations $\overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_i)}$ are calculated for every $\hat{\mathbb{H}}_i$.

$$\overleftarrow{\hat{\Upsilon}}(\mathbb{O}_1) = \overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_1)} = \hat{\Upsilon}(\hat{\nu}_2) = \{\hat{\zeta}_5, \hat{\zeta}_7\}, \quad \overleftarrow{\hat{\Upsilon}}(\mathbb{O}_2) = \overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_2)} = \hat{\Upsilon}(\hat{\nu}_6) = \{\hat{\zeta}_1, \hat{\zeta}_3\},$$
$$\overleftarrow{\hat{\Upsilon}}(\mathbb{O}_3) = \overleftarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_3)} = \hat{\Upsilon}(\hat{\nu}_8) = \{\hat{\zeta}_2, \hat{\zeta}_4\},$$
$$\overrightarrow{\hat{\Upsilon}}(\mathbb{O}_1) = \overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_1)} = \hat{\Upsilon}(\hat{\nu}_2) \cup \hat{\Upsilon}(\hat{\nu}_4) \cup \hat{\Upsilon}(\hat{\nu}_8) = \{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_4, \hat{\zeta}_5, \hat{\zeta}_6, \hat{\zeta}_7\} = \hat{\Xi} \setminus \{\hat{\zeta}_3\},$$
$$\overrightarrow{\hat{\Upsilon}}(\mathbb{O}_2) = \overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_2)} = \hat{\Upsilon}(\hat{\nu}_2) \cup \hat{\Upsilon}(\hat{\nu}_4) \cup \hat{\Upsilon}(\hat{\nu}_6) = \{\hat{\zeta}_1, \hat{\zeta}_3, \hat{\zeta}_4, \hat{\zeta}_5, \hat{\zeta}_6, \hat{\zeta}_7\} = \hat{\Xi} \setminus \{\hat{\zeta}_2\},$$
$$\overrightarrow{\hat{\Upsilon}}(\mathbb{O}_3) = \overrightarrow{\hat{\Lambda}_{\hat{\mathbb{G}}}(\hat{\mathbb{H}}_3)} = \hat{\Upsilon}(\hat{\nu}_2) \cup \hat{\Upsilon}(\hat{\nu}_4) \cup \hat{\Upsilon}(\hat{\nu}_8) = \{\hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_4, \hat{\zeta}_5, \hat{\zeta}_6, \hat{\zeta}_7\} = \hat{\Xi} \setminus \{\hat{\zeta}_3\}.$$

Thus, with these FHS approximations, two SSs $\overleftarrow{\hat{\mathbb{S}}_1} = (\overleftarrow{\hat{\Upsilon}}, \hat{\mho})$ and $\overrightarrow{\hat{\mathbb{S}}_1} = (\overrightarrow{\hat{\Upsilon}}, \hat{\mho})$ are constructed that are tabulated in Tables 12 and 13 respectively.

Currently, three FSs $\chi_{\hat{\mathbb{S}}_1}, \chi_{\overleftarrow{\hat{\mathbb{S}}_1}}, \chi_{\overrightarrow{\hat{\mathbb{S}}_1}} : \hat{\Xi} \to [0,1]$ are determined by using Eqs. 8, 9, and 10 and their values are calculated as

$$\chi_{\hat{\mathbb{S}}_1} = \{(\hat{\zeta}_1, \tfrac{1}{3}), (\hat{\zeta}_2, \tfrac{1}{3}), (\hat{\zeta}_3, \tfrac{1}{3}), (\hat{\zeta}_4, \tfrac{2}{3}), (\hat{\zeta}_5, \tfrac{2}{3}), (\hat{\zeta}_6, 0), (\hat{\zeta}_7, \tfrac{2}{3})\},$$
$$\chi_{\overleftarrow{\hat{\mathbb{S}}_1}} = \{(\hat{\zeta}_1, \tfrac{1}{3}), (\hat{\zeta}_2, \tfrac{1}{3}), (\hat{\zeta}_3, \tfrac{1}{3} \text{[illegible]} \chi_{\overrightarrow{\hat{\mathbb{S}}_1}} = \text{[illegible]} (\hat{\zeta}_4, 1), (\hat{\zeta}_5, 1), (\hat{\zeta}_6, 1), (\hat{\zeta}_7, 1)\}$$

.

A fuzzy SS $(\hat{\lambda}, \hat{F})$ is constructed with $\hat{\lambda} : \hat{F} \to \mathbb{L}^{\hat{\Xi}}$ defined by $\hat{\lambda}(LC) = \chi_{\overrightarrow{\hat{\mathbb{S}}_1}}$, $\hat{\lambda}(MC) = \chi_{\hat{\mathbb{S}}_1}$ and $\hat{\lambda}(HC) = \chi_{\overleftarrow{\hat{\mathbb{S}}_1}}$. Now assign weights $\hat{\eta}_{\chi_{\overrightarrow{\hat{\mathbb{S}}_1}}} = 0.4214$, $\hat{\eta}_{\chi_{\hat{\mathbb{S}}_1}} = 0.3234$, and $\hat{\eta}_{\chi_{\overleftarrow{\hat{\mathbb{S}}_1}}} = 0.2552$. Using Eq. 11, weighted evaluation values for all $\hat{\zeta}_j$ are computed as

$$\hat{\Upsilon}(\hat{\zeta}_1) = 0.4214 \times \hat{\lambda}(LC)(\hat{\zeta}_1) + 0.3234 \times \hat{\lambda}(MC)(\hat{\zeta}_1) + 0.2552 \times \hat{\lambda}(HC)(\hat{\zeta}_1) = 0.5998,$$
$$\hat{\Upsilon}(\hat{\zeta}_2) = 0.4214 \times \hat{\lambda}(LC)(\hat{\zeta}_2) + 0.3234 \times \hat{\lambda}(MC)(\hat{\zeta}_2) + 0.2552 \times \hat{\lambda}(HC)(\hat{\zeta}_2)$$
$$= 0.4593333,$$
$$\hat{\Upsilon}(\hat{\zeta}_3) = 0.4214 \times \hat{\lambda}(LC)(\hat{\zeta}_3) + 0.3234 \times \hat{\lambda}(MC)(\hat{\zeta}_3) + 0.2552 \times \hat{\lambda}(HC)(\hat{\zeta}_3)$$
$$= 0.3188667,$$
$$\hat{\Upsilon}(\hat{\zeta}_4) = 0.4214 \times \hat{\lambda}(LC)(\hat{\zeta}_4) + 0.3234 \times \hat{\lambda}(MC)(\hat{\zeta}_4) + 0.2552 \times \hat{\lambda}(HC)(\hat{\zeta}_4)$$
$$= 0.7075,$$
$$\hat{\Upsilon}(\hat{\zeta}_5) = 0.4214 \times \hat{\lambda}(LC)(\hat{\zeta}_5) + 0.3234 \times \hat{\lambda}(MC)(\hat{\zeta}_5) + 0.2552 \times \hat{\lambda}(HC)(\hat{\zeta}_5)$$
$$= 0.8489,$$
$$\hat{\Upsilon}(\hat{\zeta}_6) = 0.4214 \times \hat{\lambda}(LC)(\hat{\zeta}_6) + 0.3234 \times \hat{\lambda}(MC)(\hat{\zeta}_6) + 0.2552 \times \hat{\lambda}(HC)(\hat{\zeta}_6)$$
$$= 0.4214,$$
$$\hat{\Upsilon}(\hat{\zeta}_7) = 0.4214 \times \hat{\lambda}(LC)(\hat{\zeta}_7) + 0.3234 \times \hat{\lambda}(MC)(\hat{\zeta}_7) + 0.2552 \times \hat{\lambda}(HC)(\hat{\zeta}_7)$$
$$= 0.7075.$$

Therefore, the spinal cord disorder are ranked as follows based on weighted evaluation values $\hat{\zeta}_5 > \hat{\zeta}_7 = \hat{\zeta}_4 > \hat{\zeta}_1 > \hat{\zeta}_2 > \hat{\zeta}_6 > \hat{\zeta}_3$ this indicates that, nationwide, lumbar SCD is the most common kind of SCD.

Comparison and discussion

To improve the accuracy and consistency of the results, multi-argument approximation functions, and sub-parametric values must be considered when making

Table 10 Real description of FHS $\hat{\mathbb{S}}$ in tabular form

$\hat{\mathbb{S}}$	$\hat{\zeta}_1$	$\hat{\zeta}_2$	$\hat{\zeta}_3$	$\hat{\zeta}_4$	$\hat{\zeta}_5$	$\hat{\zeta}_6$	$\hat{\zeta}_7$
$\hat{\nu}_2$	0	0	0	0	0.7	0	0.5
$\hat{\nu}_4$	0.3	0	0	0.5	0	0.7	0.9
$\hat{\nu}_6$	0.2	0	0.4	0	0	0	0
$\hat{\nu}_8$	0	0.6	0	0.8	0	0	0

Table 11 Tabular representation of $\hat{\mathbb{S}}_1$

$\hat{\mathbb{S}}$	$\hat{\zeta}_1$	$\hat{\zeta}_2$	$\hat{\zeta}_3$	$\hat{\zeta}_4$	$\hat{\zeta}_5$	$\hat{\zeta}_6$	$\hat{\zeta}_7$	$\hat{\mathbf{H}}_i$
$\hat{\mathbb{O}}_1$	0	0	0	0.2	0.4	0	0.4	$\hat{\mathbf{H}}_1$
$\hat{\mathbb{O}}_2$	0.3	0	0.5	0	0	0	0.7	$\hat{\mathbf{H}}_2$
$\hat{\mathbb{O}}_3$	0	0.1	0	0.3	0.8	0	0	$\hat{\mathbf{H}}_3$

Table 12 Tabular representation for lower approximations of $\hat{\mathbb{S}}_1$

$\hat{\mathbb{S}}$	$\hat{\zeta}_1$	$\hat{\zeta}_2$	$\hat{\zeta}_3$	$\hat{\zeta}_4$	$\hat{\zeta}_5$	$\hat{\zeta}_6$	$\hat{\zeta}_7$	$\overleftarrow{\hat{\Upsilon}}(\mathbb{O}_i)$
$\hat{\mathbb{O}}_1$	0	0	0	0	o.3	0	0.8	$\overleftarrow{\hat{\Upsilon}}(\mathbb{O}_1)$
$\hat{\mathbb{O}}_2$	0.2	0	0.5	0	0	0	0	$\overleftarrow{\hat{\Upsilon}}(\mathbb{O}_2)$
$\hat{\mathbb{O}}_3$	0	0.5	0	0.8	0	0	0	$\overleftarrow{\hat{\Upsilon}}(\mathbb{O}_3)$

Table 13 Tabular representation for upper approximations of $\hat{\mathbb{S}}_1$

$\hat{\mathbb{S}}$	$\hat{\zeta}_1$	$\hat{\zeta}_2$	$\hat{\zeta}_3$	$\hat{\zeta}_4$	$\hat{\zeta}_5$	$\hat{\zeta}_6$	$\hat{\zeta}_7$	$\overrightarrow{\hat{\Upsilon}}(\mathbb{O}_i)$
$\hat{\mathbb{O}}_1$	0.9	0.3	0	0.6	0.4	0.2	0.5	$\overrightarrow{\hat{\Upsilon}}(\mathbb{O}_1)$
$\hat{\mathbb{O}}_2$	0.2	0	0.7	0.4	0.8	0.1	0.6	$\overrightarrow{\hat{\Upsilon}}(\mathbb{O}_2)$
$\hat{\mathbb{O}}_3$	0.7	0.3	0	0.4	0.6	0.2	0.5	$\overrightarrow{\hat{\Upsilon}}(\mathbb{O}_3)$

decisions. A more comprehensive analysis is made possible by sub-parametric values, which draw attention to subtle differences within each parameter that might go unnoticed if only large classes were considered. This systematic methodology facilitates the identification of minute patterns and trends that may significantly influence the decision-making process. However, multi-argument approximate functions provide a more systematic and accurate representation of the situation at hand by making it simpler to quantify elusive correlations between multiple parameters. By approximating these correlations, decision-makers can foresee outcomes, optimize solutions, and account for uncertainty more precisely. Combining these methods ensures a more thorough and precise examination, leading to more insightful and practical judgments. The hypersoft fuzzy rough technique is used in the suggested study, which improves the group's overall expert primary evaluation findings. The facilitation that was covered before in this part is provided by its hypersoft fuzzy setting. This results in a more dependable choice of the ideal object. The lower and upper approximations are useful ways to deal with uncertainty and imprecision in data processing in rough set theory. The lower approximation, which displays the set of items contained in a concept based on available data, ensures strong certainty in classification. Conversely, the upper approximation accounts for all plausible conceptual elements while accounting for potential ambiguity. Therefore, the objects that certain experts in the primary evaluation mistakenly chose as the optimal objects can be removed using the hypersoft fuzzy lower approximation, and the optimal objects that some experts in the evaluation might have missed can be added using the hypersoft fuzzy upper approximation. As a result, even though the evaluation soft set in the proposed model considers and characterizes the subjectivity of decision-making, the use of hypersoft fuzzy rough sets may, under some circumstances, automatically reduce the errors resulting from the subjectivity of the evaluation that experts provide. In recent years, a new approach to comparison has emerged in the literature, aimed at evaluating the flexibility and reliability of theoretical structures in relation to existing frameworks. This type of comparison is particularly useful when no directly comparable work exists in the current body of literature. It typically relies on significant criteria or factors to assess the theoretical structure's robustness, practical applicability, and novelty, providing a systematic way to benchmark its performance or effectiveness against relevant standards or paradigms. The same case is for this study, therefore, structural comparison is presented in Table 14 which clearly depicts the flexibility of the proposed framework.

The significant advantages of the proposed framework are outlined as:

1. It provides flexible fuzzy set arrangements to handle unclear and insufficient information related to SCD in particular and medical diagnoses, in general.
2. In the context of SCD, it employs hypersoft settings to capture vague or overlapping attribute or their respective values, which are common in complex decision-making scenarios. Moreover, its modified multi-argument based approximate mapping can support decision-making in systems with multiple

Table 14 Comparison with existing frameworks

References	Frameworks	Modelling incompleteness	Modelling uncertainties	Modelling vagueness	Multi-argument approximations	Evaluation of SCD
Molodtsov [13]	SS	Inadequate	Inadequate	Adequate	Inadequate	Inadequate
Smarandache [15]	HSS	Inadequate	Inadequate	Adequate	Adequate	Inadequate
Debnath [17]	FHS	Inadequate	Adequate	Adequate	Adequate	Inadequate
Pawlak [24]	RS	Adequate	Inadequate	Inadequate	Inadequate	Inadequate
Kamacı [34]	Hypersoft RS	Adequate	Inadequate	Adequate	Adequate	Inadequate
Rahman et al. [35]	Fuzzy hypersoft RS	Adequate	Adequate	Adequate	Adequate	Inadequate
Abdullah et al. [36]	Hypersoft RS	Adequate	Inadequate	Adequate	Adequate	Inadequate
Rahimi et al. [42]	FS	Inadequate	Adequate	Inadequate	Inadequate	Adequate
Surdilovic [50]	FS	Inadequate	Adequate	Inadequate	Inadequate	Adequate
Proposed study	HFRS	Adequate	Adequate	Adequate	Adequate	Adequate

interdependent attributes that may change over time. In this way, it yields reliable and unbiased quick decisions.

3. Its modified lower and upper approximations can classify boundary regions effectively in the context of data that do not fit neatly into predefined categories. The lower approximation ensures a precise representation of elements that unequivocally belong to the set, enhancing reliability in uncertain contexts. In contrast, the upper approximation includes elements with partial or potential membership, capturing the nuances of borderline cases. This dual-layered approach allows for a comprehensive analysis of ambiguous data, ensuring adaptability and improved accuracy in scenarios involving imprecise, incomplete, or conflicting information.

Conclusion

Accurately diagnosing spinal cord disorder (SCD) is still very challenging because the facts surrounding the condition are complex and often obscure. Conventional diagnostic methods frequently fall short, requiring expensive and extensive testing. The integration of sophisticated frameworks such as machine learning, data mining, and MADM is one potential method to increase DM and diagnostic efficiency. In order to successfully address the complexity of SCD diagnosis, this research introduces a unique method using HFRS, which blends RS and FS theories. Case studies show how this framework can greatly increase the precision of diagnosis and planning of treatment. According to the results, medical practitioners may profit substantially from using HFRS in SCD diagnosis as it provides a reliable instrument for managing complexity and uncertainty in medical data. In addition to improving the field of medical diagnostics, this strategy may find wider use in other areas where MADM under uncertainty is necessary. Due to their potential complexity and processing overhead, multi-argument-based parameters in FHS are the study's main limitations. These parameters may hinder real-time applications, particularly when dealing with large datasets. The algorithm's robustness may also be questioned in cases where the criteria or sub-criteria are ambiguous or overlap. Future research could explore the extension of FHS to address other complex medical conditions beyond SCD, enabling broader applicability in healthcare diagnostics. Additionally, the integration of machine learning techniques with the proposed framework could enhance its adaptability and precision in handling large-scale medical data. Investigating the real-time implementation of the algorithm in clinical settings and integrating it with advanced imaging technologies and electronic health records could further improve diagnostic accuracy and decision-making efficiency. Finally, a comparative analysis with alternative intelligent diagnostic systems could validate and refine the proposed approach, establishing its place as a benchmark in medical decision support systems.

Acknowledgements
Not applicable.

Authors' contributions
Conceptualization, M.A., and A.U.R.; methodology, M. A., K.A.K., and A.U.R.; software, M. A., and A.U.R.; validation, M. A., K.A.K., and A.U.R.; formal analysis, M.A., and K.A.K.; investigation, M. A., K.A.K., and A.U.R.; visualization, M.A., and A.U.R.; supervision, K.A.K., and A.U.R. All authors have read and agreed to the published version of the manuscript.

Funding
The authors received no specific funding for this study.

Data availability
All data generated or analyzed during this study are included in this published article.

Declarations

Ethics approval and consent to participate
Not applicable.

Consent for publication
Not applicable.

Competing interests
The authors declare no competing interests.

Received: 17 October 2024 Accepted: 21 February 2025
Published online: 10 March 2025

References

1. Center National Spinal Cord Injury Statistical. Facts and Figures at A Glance. Birmingham: University of Alabama at Birmingham; 2017.
2. Karsy M, Hawryluk G. Modern medical management of spinal cord injury. Curr Neurol Neurosci Rep. 2019;19:1–7. https://doi.org/10.1007/s11910-019-0984-1.
3. Wyndaele M, Wyndaele JJ. Incidence, prevalence and epidemiology of spinal cord injury: what learns a worldwide literature survey? Spinal Cord. 2006;44(9):523–9. https://doi.org/10.1038/sj.sc.3101893.
4. DeVivo MJ. Epidemiology of traumatic spinal cord injury: trends and future implications. Spinal Cord. 2012;50(5):365–72. https://doi.org/10.1038/sc.2011.178.
5. Jain A, Brooks JT, Rao SS, Ain MC, Sponseller PD. Cervical fractures with associated spinal cord injury in children and adolescents: epidemiology, costs, and in-hospital mortality rates in 4418 patients. J Child Orthop. 2015;9(3):171–5. https://doi.org/10.1007/s11832-015-065.
6. Sherrod B, Karsy M, Guan J, Brock AA, Eli IM, Bisson EF, Dailey AT. Spine trauma and spinal cord injury in Utah: a geographic cohort study utilizing the National Inpatient Sample. J Neurosurg Spine. 2019;31(1):93–102. https://doi.org/10.3171/2018.12.SPINE18964.
7. Ning GZ, Yu TQ, Feng SQ, Zhou XH, Ban DX, Liu Y, Jiao XX. Epidemiology of traumatic spinal cord injury in Tianjin. China Spinal Cord. 2011;49(3):386–90. https://doi.org/10.1038/sc.2010.130.
8. Li J, Liu G, Zheng Y, Hao C, Zhang Y, Wei B, et al. The epidemiological survey of acute traumatic spinal cord injury (ATSCD) of 2002 in Beijing municipality. Spinal Cord. 2011;49(7):777–82. https://doi.org/10.1038/sc.2011.8.
9. Eli I, Lerner DP, Ghogawala Z. Acute traumatic spinal cord injury. Neurol Clin. 2021;39(2):471–88. https://doi.org/10.1016/j.ncl.2021.02.004.
10. Orr MB, Gensel JC. Spinal cord injury scarring and inflammation: therapies targeting glial and inflammatory responses. Neurotherapeutics. 2018;15(3):541–53. https://doi.org/10.1007/s13311-018-0631-6.
11. Siddiqui AM, Khazaei M, Fehlings MG. Translating mechanisms of neuroprotection, regeneration, and repair to treatment of spinal cord injury. Prog Brain Res. 2015;218:15–54. https://doi.org/10.1016/bs.pbr.2014.12.007.
12. Zadeh LA. Fuzzy sets. Inf Control. 1965;8(3):338–53. https://doi.org/10.1016/S0019-9958(65)90241-X.
13. Molodtsov D. Soft set theory first results. Comput Math Appl. 1999;37(4–5):19–31. https://doi.org/10.1016/S0898-1221(99)00056-5.
14. Maji PK, Biswas RK, Roy A. Fuzzy soft sets. J Fuzzy Math. 2001;9(3):589–602.
15. Smarandache F. Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. Neutrosophic Sets Syst. 2018;22:168–70. https://doi.org/10.5281/zenodo.2159755.
16. Saeed M, Rahman AU, Ahsan M, Smarandache F. Theory of Hypersoft Sets: Axiomatic Properties, Aggregation Operations, Relations. Functions and Matrices Neutrosophic Sets Syst. 2022;51:744–65. https://doi.org/10.5281/zenodo.7135413.
17. Debnath S. Fuzzy hypersoft sets and its weightage operator for decision making. J Fuzzy Ext Appl. 2021;2(2):163-170. https://doi.org/10.22105/jfea.2021.275132.1083.
18. Rahman AU, Saeed M, Mohammed MA, Jaber MM, Garcia-Zapirain B. A novel fuzzy parameterized fuzzy hypersoft set and riesz summability approach based decision support system for diagnosis of heart diseases. Diagnostics. 2022;12(7):1546. https://doi.org/10.3390/diagnostics12071546.
19. Arshad M, Saeed M, Rahman AU, Zebari DA, Mohammed MA, Al-Waisy AS, Albahar M, Thanoon M. The assessment of medication effects in omicron patients through madm approach based on distance measures of interval-valued fuzzy hypersoft set. Bioengineering. 2022;9(11):706. https://doi.org/10.3390/bioengineering9110706.
20. Rahman AU, Saeed M, Mohammed MA, Abdulkareem KH, Nedoma J, Martinek R. An innovative mathematical approach to the evaluation of susceptibility in liver disorder based on fuzzy parameterized complex fuzzy hypersoft set. Biomed Signal Process Control. 2023;86:105204. https://doi.org/10.1016/j.bspc.2023.105204.
21. Xiao F. A distance measure for intuitionistic fuzzy sets and its application to pattern classification problems. IEEE Trans Syst Man Cybern Syst. 2019;51(6):3980–92. https://doi.org/10.1109/TSMC.2019.2958635.
22. Xiao F. EFMCDM: Evidential fuzzy multicriteria decision making based on belief entropy. IEEE Trans Fuzzy Syst. 2019;28(7):1477–91. https://doi.org/10.1109/TFUZZ.2019.2936368.
23. Yang Y, Su X, Zhao B, Li G, Hu P, Zhang J, Hu L. Fuzzy-based deep attributed graph clustering. IEEE Trans Fuzzy Syst. 2023;32(4):1951–64. https://doi.org/10.1109/TFUZZ.2023.3338565.
24. Pawlak Z. Rough sets. Int J Comput Inf Sci. 1982;11:341–56. https://doi.org/10.1007/BF01001956.
25. Pawlak Z. Rough set theory and its applications to data analysis. Cybern Syst. 1998;29(7):661–88. https://doi.org/10.1080/019697298125470.
26. Pawlak Z. Rough set theory and its applications. J Telecommun Inf Technol. 2002;3:7-10. https://doi.org/10.26636/jtit.2002.140.
27. Pawlak Z, Skowron A. Rudiments of rough sets. Inf Sci. 2007;1(177):3–27. https://doi.org/10.1016/j.ins.2006.06.003.
28. Dubois D, Prade H. Rough fuzzy sets and fuzzy rough sets. Int J Gen Syst. 1990;17(2–3):191–209. https://doi.org/10.1080/03081079008935107.
29. Radzikowska AM, Kerre EE. A comparative study of fuzzy rough sets. Fuzzy Sets Syst. 2002;126(2):137–55. https://doi.org/10.1016/S0165-0114(01)00032-X.
30. Ali MI. A note on soft sets, rough soft sets and fuzzy soft sets. Appl Soft Comput. 2011;11(4):3329–32. https://doi.org/10.1016/j.asoc.2011.01.003.
31. Meng D, Zhang X, Qin K. Soft rough fuzzy sets and soft fuzzy rough sets. Comput Math Appl. 2011;62(12):4635–45. https://doi.org/10.1016/j.camwa.2011.10.049.
32. Sun B, Ma W. Soft fuzzy rough sets and its application in decision making. Artif Intell Rev. 2014;41:67–80. https://doi.org/10.1007/s10462-011-9298-7.
33. Hu Q, An S, Yu D. Soft fuzzy rough sets for robust feature evaluation and selection. Inf Sci. 2010;180(22):4384–400. https://doi.org/10.1016/j.ins.2010.07.010.
34. Kamacı H. On hybrid structures of hypersoft sets and rough sets. Int J Mod Sci Technol. 2021;6(4):69–82.
35. Rahman AU, Saeed M, Mohammed MA, Majumdar A, Thinnukool O. Supplier selection through multicriteria decision-making algorithmic approach based on rough approximation of fuzzy hypersoft sets for construction project. Buildings. 2022;12(7):940. https://doi.org/10.3390/buildings12070940.
36. Abdullah M, Khan KA, Frnda J, Rahman AU. A novel algorithmic multi-attribute decision-making framework for the evaluation of energy systems using rough approximations of hypersoft sets. Heliyon. 2024;10(23):e40592. https://doi.org/10.1016/j.heliyon.2024.e40592.
37. Namburu A, kumar Samay S, Edara SR. Soft fuzzy rough set-based MR brain image segmentation. Appl Soft Comput. 2017;54:456-466. https://doi.org/10.1016/j.asoc.2016.08.020.
38. Xing J, Gao C, Zhou J. Weighted fuzzy rough sets-based tri-training and its application to medical diagnosis. Appl Soft Comput. 2022;124:109025. https://doi.org/10.1016/j.asoc.2022.109025.
39. Sanabria J, Rojo K, Abad F. A new approach of soft rough sets and a medical application for the diagnosis of Coronavirus disease. AIMS Math. 2023;8(2):2686–707. https://doi.org/10.3934/math.2023141.
40. El-Bably MK, Abu-Gdairi R, El-Gayar MA. Medical diagnosis for the problem of Chikungunya disease using soft rough sets. AIMS Math. 2023;8(4):9082–105. https://doi.org/10.3934/math.2023455.
41. Das S, Sil J. Managing boundary uncertainty in diagnosing the patients of rural area using fuzzy and rough set. J Healthc Inform Res. 2022;6(1):1–47. https://doi.org/10.1007/s41666-021-00109-4.
42. Rahimi Damirchi-Darasi S, Fazel Zarandi MH, Turksen IB, Izadi M. Type-2 fuzzy rule-based expert system for diagnosis of spinal cord disorders. Sci Iran. 2019;26(1):455-471. https://doi.org/10.24200/SCI.2018.20228.
43. Bose A, Maulik U, Sarkar A. An entropy-based membership approach on type-II fuzzy set (EMT2FCM) for biomedical image segmentation. Eng

Appl Artif Intell. 2024;127:107267. https://doi.org/10.1016/j.engappai.2023.107267.
44. Mago VK, Mehta R, Woolrych R, Papageorgiou EI. Supporting meningitis diagnosis amongst infants and children through the use of fuzzy cognitive mapping. BMC Med Inform Dec Making. 2012;12:1–12. https://doi.org/10.1186/1472-6947-12-98.
45. Feng F, Li C, Davvaz B, Ali MI. Soft sets combined with fuzzy sets and rough sets: a tentative approach. Soft Comput. 2010;14:899–911. https://doi.org/10.1007/s00500-009-0465-6.
46. Feng F. Soft rough sets applied to multicriteria group decision making. Ann Fuzzy Math Inform. 2011;2(1):69–80.
47. Athanasiou M, Clark JY. A Bayesian network model for the diagnosis of the caring procedure for wheelchair users with spinal injury. Comput Methods Prog Biomed. 2009;95(2):S44–54. https://doi.org/10.1016/j.cmpb.2009.02.018.
48. Pradeep S, Arulselvan S, Deepak A, Gokul N. Detection of Spinal Cord Injury using Deep Learning Algorithm. In: 2022 International Conference on Sustainable Computing and Data Communication Systems (ICSCDS). IEEE; 2022. pp. 270-275. https://doi.org/10.1109/ICSCDS53736.2022.9760935.
49. Jasim KM, Brindha T. Injury classification and level detection of the spinal cord based on the optimized recurrent neural network. Bio-Algoritm Med-Syst. 2021;17(1):25–40.
50. Surdilovic T. Fuzzy mouse cursor control system for computer users with spinal cord injuries. 2006. https://doi.org/10.57709/1059394.
51. Guerra TM, Blandeau M, Nguyen AT, Srihi H, Dequidt A. Stabilizing unstable biomechanical model to understand sitting stability for persons with spinal cord injury. IFAC-PapersOnLine. 2020;53(2):8001–6. https://doi.org/10.1016/j.ifacol.2020.12.2225.

Publisher's Note